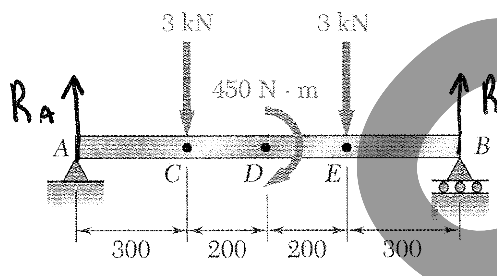
Name: Solution
ID:
Section:

Exam II

Monday May 25, 2009

All work **must** be shown to receive full credit.

Problem 1. (25 pts.) Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.



• $\Sigma M_B = 0$

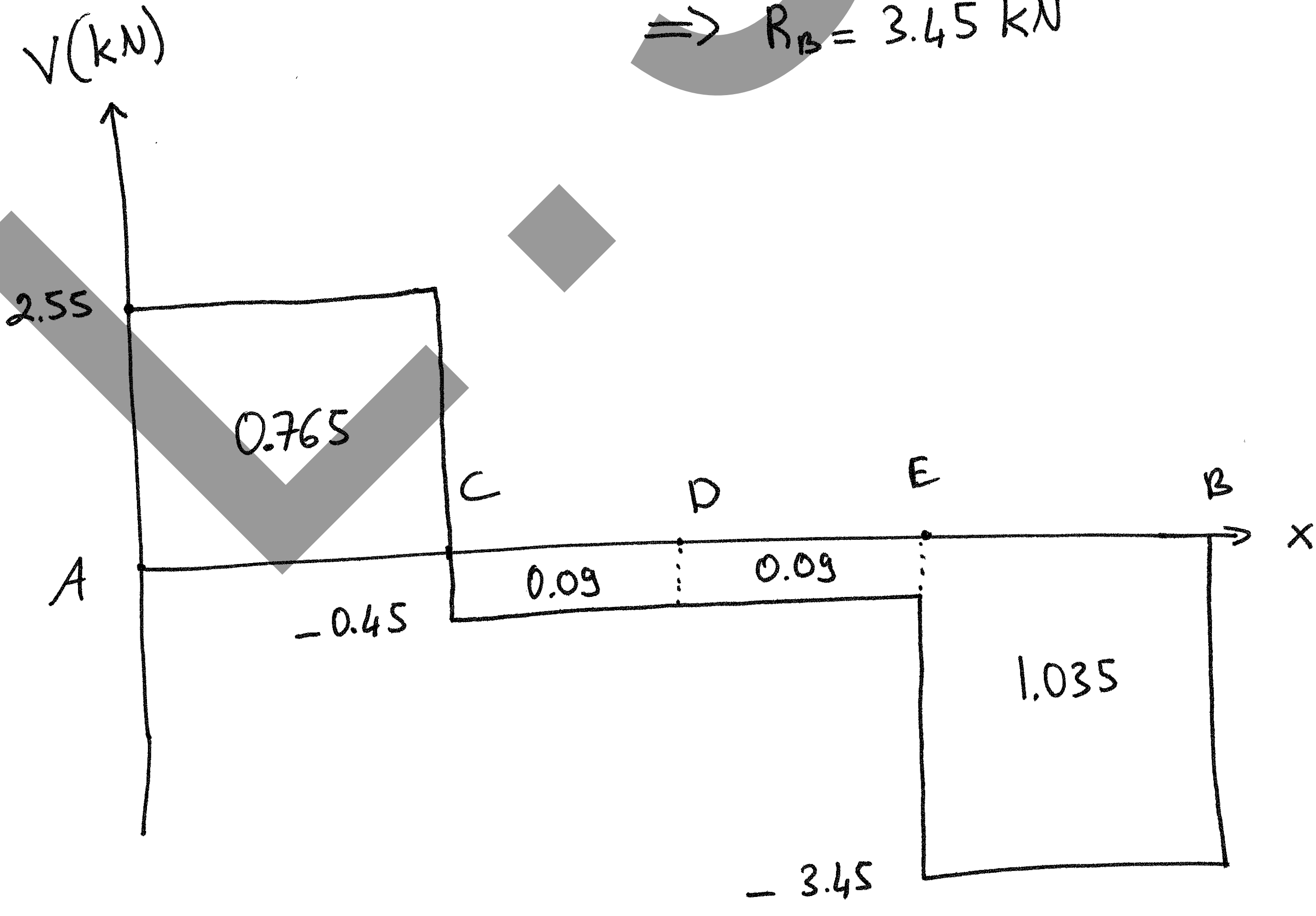
$3000 \times 0.7 + 3000 \times 0.3 - 450 - R_A \times 1 = 0$

$\Rightarrow R_A = 2.55 \text{ kN}$

• $\Sigma M_A = 0$

$3000 \times 0.3 + 3000 \times 0.7 + 450 + R_B \times 1 = 0$

$\Rightarrow R_B = 3.45 \text{ kN}$



$M_A = 0$

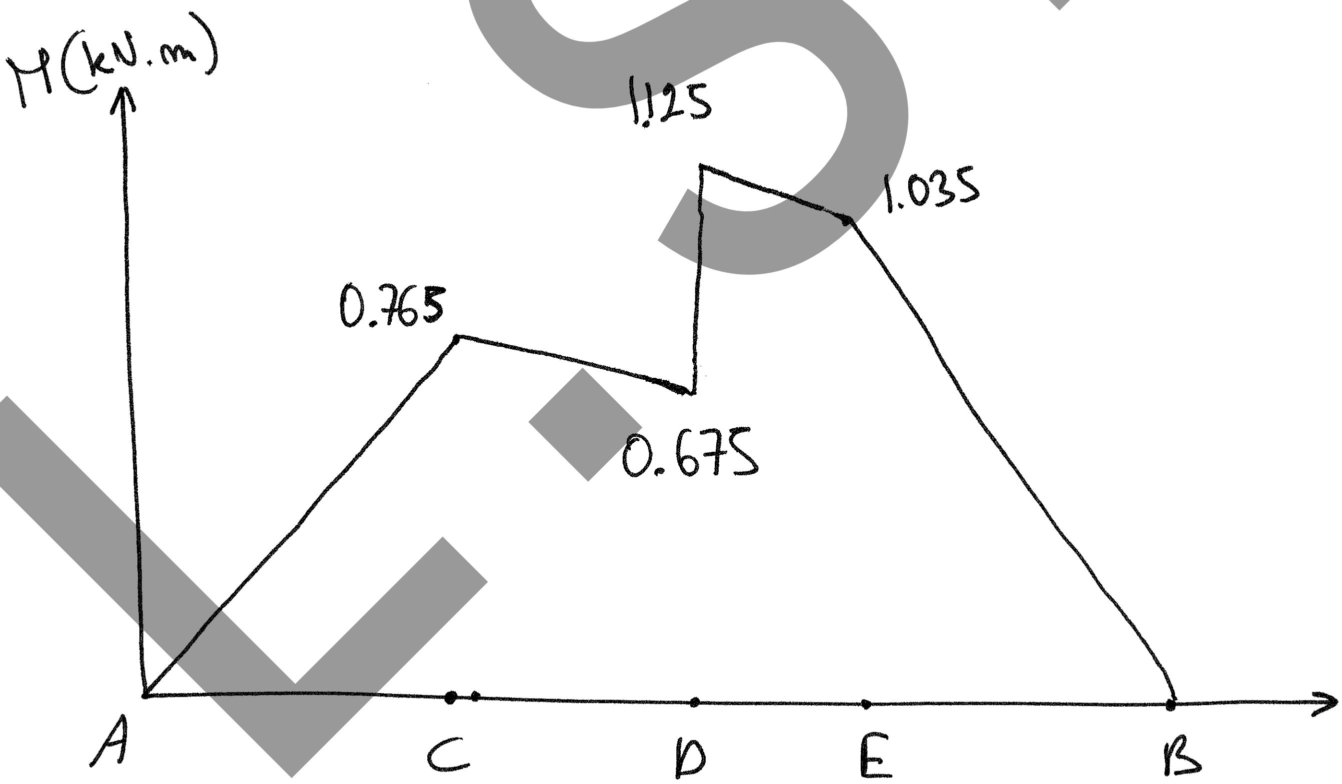
$M_C = M_A + 0.765 = 0.765$

$M_{D^-} = M_C - 0.09 = 0.675$

$M_{D^+} = M_{D^-} + 0.45 = 1.125$

$M_E = M_{D^+} - 0.09 = 1.035$

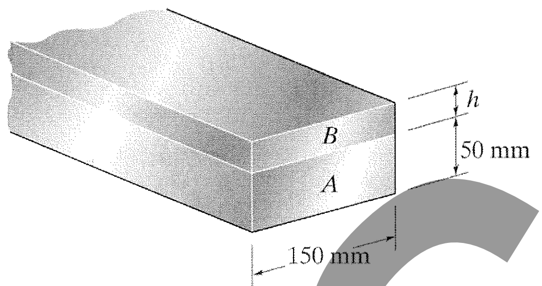
$M_B = 0$

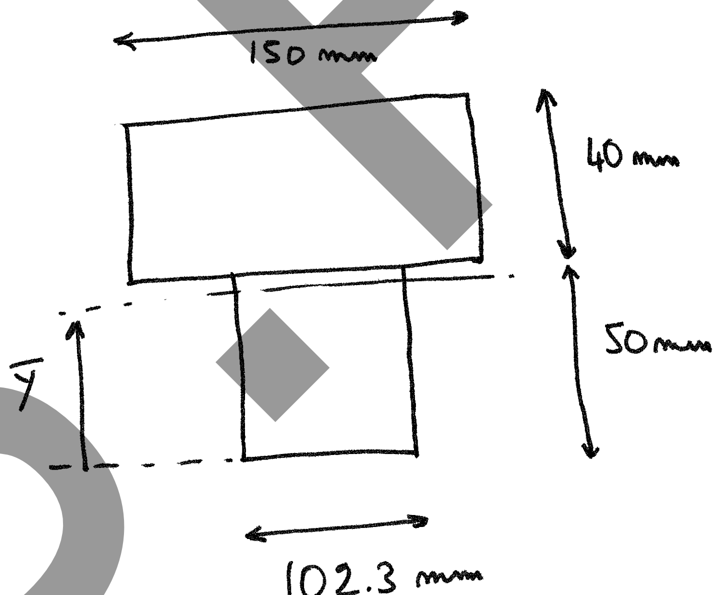


Maximum $|V| = 3.45$ kN

Maximum $|M| = 1.125$ kN.m

Problem 2. (25 pts.) The composite beam is made of Aluminum (A) and brass (B). If the height $h = 40\ mm$, determine the maximum moment that can be applied to the beam if the allowable bending stress for the aluminum is $(\sigma_{all})_A = 128\ MPa$ and for the brass $(\sigma_{all})_B = 35\ MPa$. ($E_A = 68.9\ GPa$ and $E_B = 101.0\ GPa$)





$$n = \frac{E_A}{E_B} = \frac{68.9}{101} = 0.682$$

$$b_A = n\, b_B = 0.1023 m$$

$$\bar{y} = \frac{102.3 \times 50 \times 25 + 150 \times 40 \times 70}{102.3 \times 50 + 150 \times 40} = 49.3\ mm$$

$$I = I_A + I_B$$

$$= \frac{1}{12}\, 102.3 \times 50^3 + 102.3 \times 50 \times 24.3^2 + \frac{1}{12}\, 150 \times 40^3 + 150 \times 40 \times 20.7^2$$

$$I = 7.45 \times 10^{-6}\ m^4$$

- Failure in brass : B

$$\sigma_{all_B} = \frac{M_B c_b}{I} \qquad c_b = 0.0407\,m$$

$$\Longrightarrow \quad M_B = \frac{35\times10^{6}\times7.45\times10^{-6}}{0.0407} = 6.4\ kN.m$$

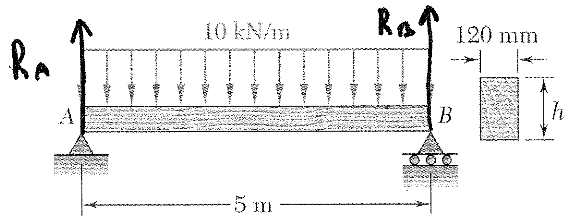
- Failure in Aluminum : A

$$\sigma_{all_A} = n\frac{M_A c_A}{I} \Longrightarrow \quad c_A = 0.0493\ m$$

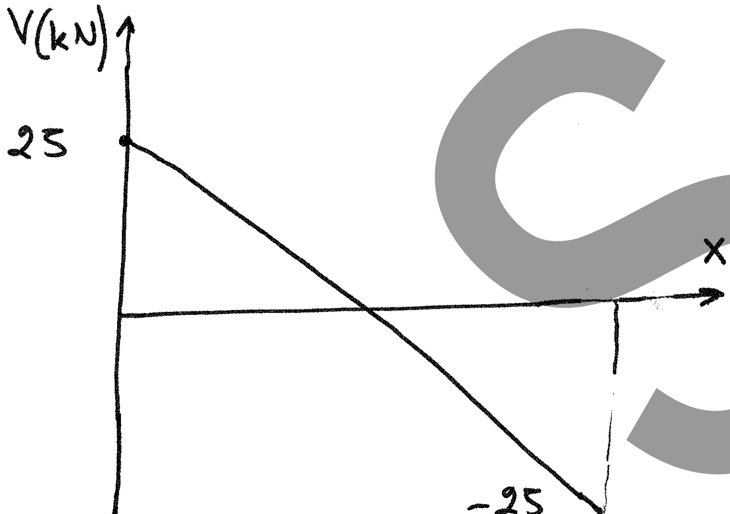
$$\Longrightarrow \quad M_A = \frac{128\times10^{6}\times7.45\times10^{-6}}{0.682\times0.0493} = 28.3\ kN.m$$

$$\boxed{M_{max} = 6.4\ kN.m}$$

Problem 3. (25 pts.) The timber beam is designed to support the distributed loading shown. Knowing that for the grade of timber used $\tau_{all} = 0.8\ MPa$, determine the minimum required depth h of the beam.

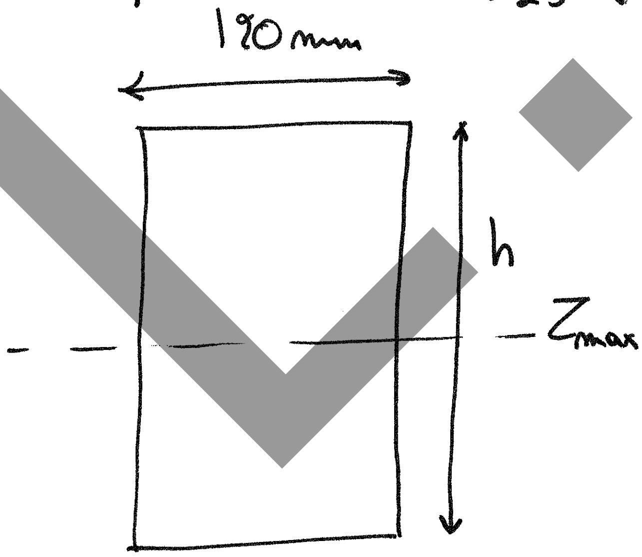


$$R_A = R_B = 25\ kN$$



Maximum V occurs at A or B

$$|V_{max}| = 25\ kN$$



$$\tau_{max} = \frac{V_{max} Q}{I t} = \tau_{all}$$

$$Q = \frac{h}{2} \times 0.12 \times \frac{h}{4} = 0.015 h^2$$

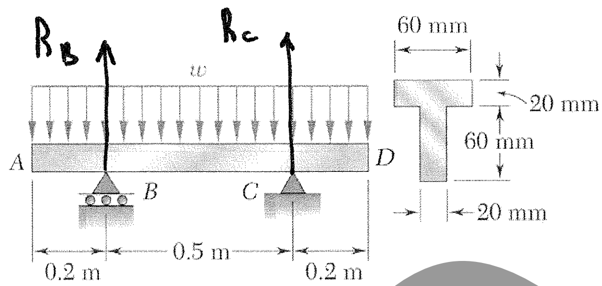
$$t = 0.12$$

$$I = \frac{1}{12} 0.12 \times h^3$$

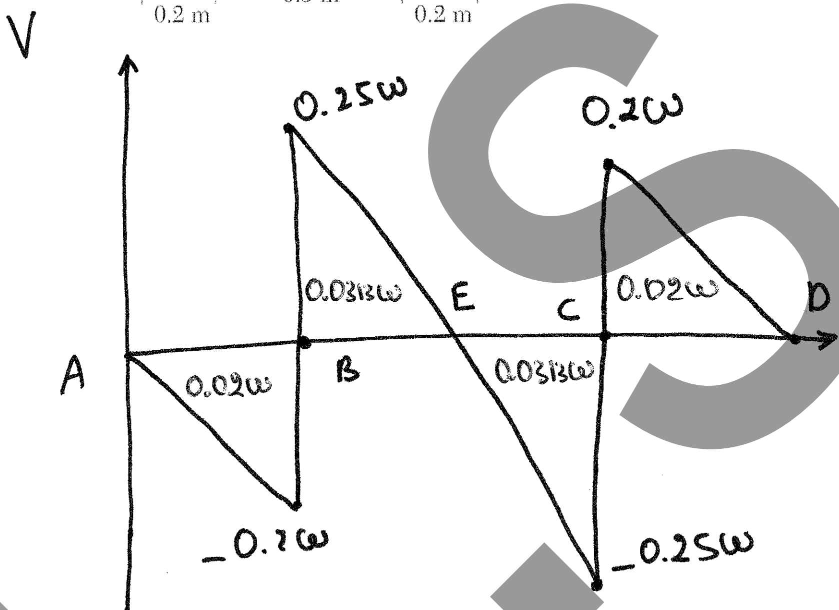
$$\Rightarrow 0.8 \times 10^6 = \frac{25 \times 10^3 \times 0.015 h^2}{\frac{1}{12} \times 0.12^2 \times h^3} \Rightarrow h = 0.3906$$

The minimum required depth h is $h_{min} = 390.6$ mm

Problem 4. (25 pts.) Determine the largest permissible distributed load w for the beam shown, knowing that the allowable normal stress is $+84\ MPa$ in tension and $-205\ MPa$ in compression.



$R_B = R_c = 0.45w$



$V_A = 0$

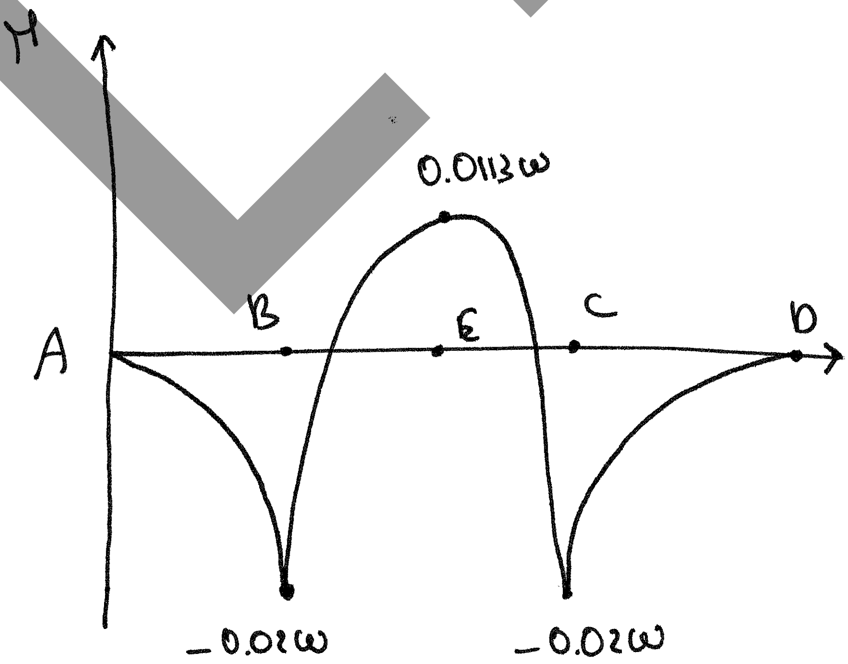
$V_B^- = V_A - 0.2w = -0.2w$

$V_B^+ = V_B^- + R_B = 0.25w$

$V_C^- = V_B^+ - 0.5w = -0.25w$

$V_C^+ = V_C^- + R_c = 0.2w$

$V_D = 0$



$M_A = 0$

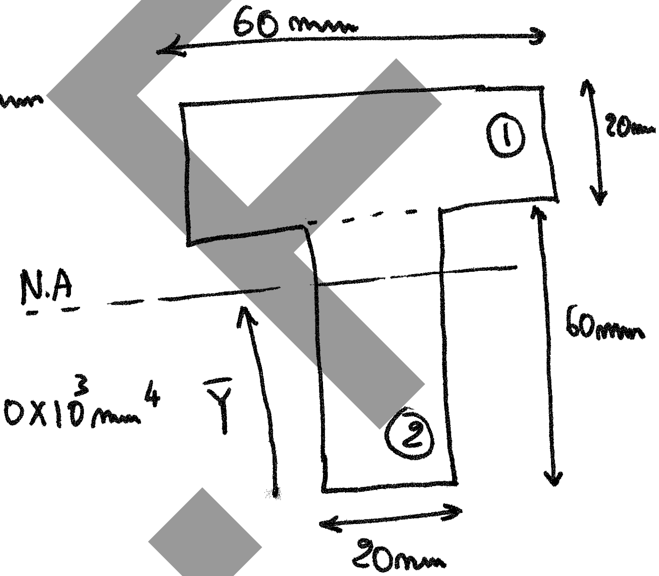
$M_B = M_A \quad 0.02w = -0.02w$

$M_E = M_B + 0.0313w = 0.0113w$

$M_C = M_E - 0.0313w = -0.02w$

$M_D = 0$

$$\bar{Y} = \frac{60\times20\times30 + 60\times20\times70}{60\times20 + 60\times20} = 50\ \text{mm}$$



$$I = I_1 + I_2$$

$$= \frac{1}{12}\,20\times60^3 + 20\times60\times20^2$$

$$+ \frac{1}{12}\,60\times20^3 + 60\times20\times20^2 = 1360\times10^3\ \text{mm}^4$$

$$C_{top} = 30\ \text{mm}$$

$$C_{bottom} = 50\ \text{mm}$$

• At B and C : $M = -0.02w$

Tension at top $\quad \sigma_{all} = \dfrac{M C_{top}}{I} \Rightarrow M = \dfrac{\sigma_{all_T} I}{C_{top}} = 0.02w$

$$\Rightarrow w = \frac{84\times10^6\times1360\times10^{-9}}{0.02\times0.03} = 190.4\times10^3\ \text{N/m}$$

Compression at bottom $\Rightarrow w = \dfrac{\sigma_{all_C} I}{0.02\times C_{bottom}} = \dfrac{205\times10^6\times1360\times10^{-9}}{0.02\times0.05}$

$$\Rightarrow w = 278.8\times10^3\ \text{N/m}$$

• At E : $M = 0.0113w$

Tension at bottom $\Rightarrow w = \dfrac{\sigma_{all_T} I}{0.02\, C_{bottom}} = \dfrac{84\times10^6\times1360\times10^{-9}}{0.0113\times0.05} = 202\times10^3\ \text{N/m}$

Compression at Top $\Rightarrow w = \dfrac{\sigma_{all_C} I}{0.02\times C_{top}} = \dfrac{205\times10^6\times1360\times10^{-9}}{0.0113\times0.03} = 826\times10^3\ \text{N/m}$

$$\Rightarrow \boxed{w_{max} = 190.4\ \text{kN/m}}$$